# Differential Geometry

## B. Math. III

## Mid-Term Examination

**Instructions:** All questions carry equal marks.

1. Define parametrized curve, regular parametrized curve and unit-speed parametrized curve. Prove that a parametrized curve has a unit-speed reparametrization if and only if it regular. Give a unit speed parametrization of the circle of radius 5 around the point $(1, 0)$ in the plane.

2. Let $C$ be a curve in $\mathbb{R}^3$ given by the parametrization

   $$\gamma(t) = (\cos t, \sin t, 3t).$$

   Draw $C$ and find its curvature.

3. Let $C$ be a regular curve in space with nowhere vanishing curvature. Define its torsion and prove that $C$ is a plane curve if and only if its torsion is zero at all points.

4. Define a smooth surface in $\mathbb{R}^3$, explaining every term used in its definition. Prove that the set of unit norm vectors in $\mathbb{R}^3$ is a smooth surface.

5. Define the standard unit normal of a regular surface patch. If $\sigma(u, v)$ is a surface patch with standard unit normal $N$, and the First Fundamental Form $(E, F, G)$, then prove that

   $$N \times \sigma_u = \frac{E\sigma_v - F\sigma_u}{\sqrt{EG - F^2}}$$